INTRODUCTION TO MATHEMATICS FOR BUSINESS AND ECONOMICS
Entrance Qualifications

SPM 0 INTRODUCTION

The course *Introduction to Mathematics for Business and Economics* (module ALMAT01) assumes that students are at Swiss Professional Maturity (SPM)-level (Berufsmaturitätsniveau). Mathematical terms, concepts and theories associated with that level of competence will not be systematically repeated.

This entry test helps you evaluate your mathematical competences. In case you should not be able to solve most of the problems, it is recommended to work through the *Preparatory Course* provided on www.emath.hslu.ch

One of the many books that might be helpful is Ian Jacques: Mathematics for Economics and Business. Pearson, 9th edition.

Hochschule Luzern
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SPM 1 SETS

Exercise SPM 1-1

Let sets A, B, C, and D be defined as follows:
A people with bloodgroup A,
B female people,
C people aged at least 10,
D people in retirement.

a) Describe the following sets using the symbols A through D:
    i)   Set of retired men.
    ii)   Set of retired women having bloodgroup A.
    iii) Set of retired women having a bloodgroup other than A.

b) Describe the following sets by naming their properties:
    iv) $B \cap C$
    v) $A \cap D \setminus B$

SPM 2 ASSERTIONS

Exercise SPM 2-1

Which of the following assertions are true:

a) 10 is a natural number.
b) 10 is a rational number.
c) $\frac{1}{3}$ is a natural number.
d) $\frac{1}{3}$ is a real number.
e) π is a rational number.
f) π is a real number.

Exercise SPM 2-2

Which of the following assertions are true:

a) 10 is a natural number and smaller than 20.
b) 10 is a natural number and smaller than 9.
c) $\frac{1}{3}$ is a natural number and smaller than 20.

SPM 3 PROPOSITIONAL FORMULAS

Exercise SPM 3-1

« *x is smaller than* 10 » is a propositional formula. For which of the following numbers x is the resulting assertion true:

a) $x = 10$

b) $x = 9$

c) $x = \pi$.

Exercise SPM 3-1

« *x is a natural number and larger than* 100 » is a propositional formula. For which of the following numbers x is the resulting assertion true:

a) $x = 100$

b) $x = 101$

c) $x = 300 + \pi$.

SPM 4.1 ALGEBRAIC LAWS

Exercise SPM 4.1-1

Complete the following equations without using parantheses:

a) $x(wz - av) =$

b) $(34x + 17y)y =$

Exercise SPM 4.1-2

Complete the following equations by factoring out:

a) $abx + avx =$

b) $5xy - 10xz =$

Exercise SPM 4.1-3

Complete the following equations using just one fraction bar:

a) $\dfrac{ab}{c} - \dfrac{cd}{a} =$

b) $\dfrac{2(a+b)}{3c} + \dfrac{3(a-b)}{2a} =$

Exercise SPM 4.1-4

Complete the following equations using just one fraction bar:

a) $\dfrac{\frac{4c}{ab}}{\frac{3d}{c^2}} =$

b) $\dfrac{\frac{2(a+b)}{3c}}{\frac{3(a-b)}{2a}} =$

Exercise SPM 4.1-5

Complete the following equations by expanding the left side term, that is without using a parenthesis:

a) $(x-1)^2 =$

b) $(z+1)^3 =$

Exercise SPM 4.1-6

Complete the following equations using just one exponent:

a) $(x-1)^2 (x+1)^2 =$

b) $\dfrac{(z+1)^3}{(z-1)^3} =$

Exercise SPM 4.1-6

Name the following quantities using the units shown in brackets:

a) $1{,}300 \text{ mm}^2 \ [\text{in m}^2]$

b) $23 \ \dfrac{m}{\text{sec}} \ [\text{in } \dfrac{\text{km}}{\text{h}}]$

c) $45 \ m^3 \ [\text{in } cm^3]$

Exercise SPM 4.1-7

Complete the following equations using the log sign just once:

a) $2\log_a x + \log_a y =$

b) $3\log_a x - \dfrac{1}{3}\log_a x =$

SPM 4.2 EQUIVALENCE OF EQUATIONS

Exercise SPM 4.2-1

In which of the following pairs of equations are both equations equivalent:

a) $(x-1)(x+1) = 0 \overset{?}{\Leftrightarrow} x = 1$ or $x = -1$

b) $x^2 + x - 2 = 0 \overset{?}{\Leftrightarrow} x = 1$

SPM 4.3 TRANSFORMING EQUATIONS

Exercise SPM 4.3 - 1

Solve the following equation for y (given $y > 0$ and $x \notin \{-1,\ 1\}$):

$$\frac{\log_{10} y}{(x^2 - 1)} - 5 = 7$$

Exercise SPM 4.3-2

Determine the solutions of the following equation, that is name all real numbers satisfying them:

$$(x^2 - 3x + 9)(x^2 - 1) = 0$$

Exercise SPM 4.3-3

Determine the solutions of the following equation:

$$\frac{(x-1)(x+9)(x-9)}{(x-9)(x-17)} = 0$$

Exercise SPM 4.3-4

Determine the solutions of the following equation:

$$2^y = 2^{y+1} - 64$$

Exercise SPM 4.3-5

Determine the solutions of the following equation:

$$(x-3)^4 = 16$$

Exercise SPM 4.3-6

Determine the solutions of the following equation:

$$2^{3x-5} = 4^{5x}$$

SPM 4.4 SPECIAL EQUATIONS WITH ONE VARIABLE

Exercise SPM 4.4-1

How many real solutions does each of the following equations have:

a) $x^2 - 2x + 1 = 0$

b) $x^2 - 2x + 1 = 1$

c) $x^2 - 2x + 1 = -1$

Exercise SPM 4.4-2

What are the parameter values a and b that make the line defined by $y = ax + b$ meet the following requirements:

$y(0) = 2$ and $y(2) = 8$.

SPM 4.5 EQUATION SYSTEMS

Exercise SPM 4.5-1

Determine all solutions of the following equation system:

$$\left| \begin{array}{l} -2x + y = 1 \\ x + 4y = 2 \end{array} \right.$$

Exercise SPM 4.5-2

Determine all real solutions of the following equation system:

$$\left| \begin{array}{l} -2x + y = 1 \\ y^2 = -22 \end{array} \right.$$

Exercise SPM 4.5-3

Determine all solutions of the following equation system:

$$\left| \begin{array}{l} 2x + 3y = 1 \\ y + 3z = 2 \\ x + z = 1 \end{array} \right.$$

SPM 5 FUNCTIONS

Exercise SPM 5-1

Let the function f be defined by the assignment $f(x) = \sqrt{x-3}$. What is the domain of that function, that is for which numbers is this function defined?

Exercise SPM 5-2

The public transport system of the city of Aakebrö has a unit ticket price: one ticket (price: 3 kroners) entitles you to travel as far and as often as you wish by trolleybus for one day. Let C be the function showing transportation cost per day [in kroner] as a function of travelling distance [in km] per day for one person.

a) What is the domain of that function?
b) What is the assignment of that function?
c) Sketch the graph of that function in the coordinate cross below.
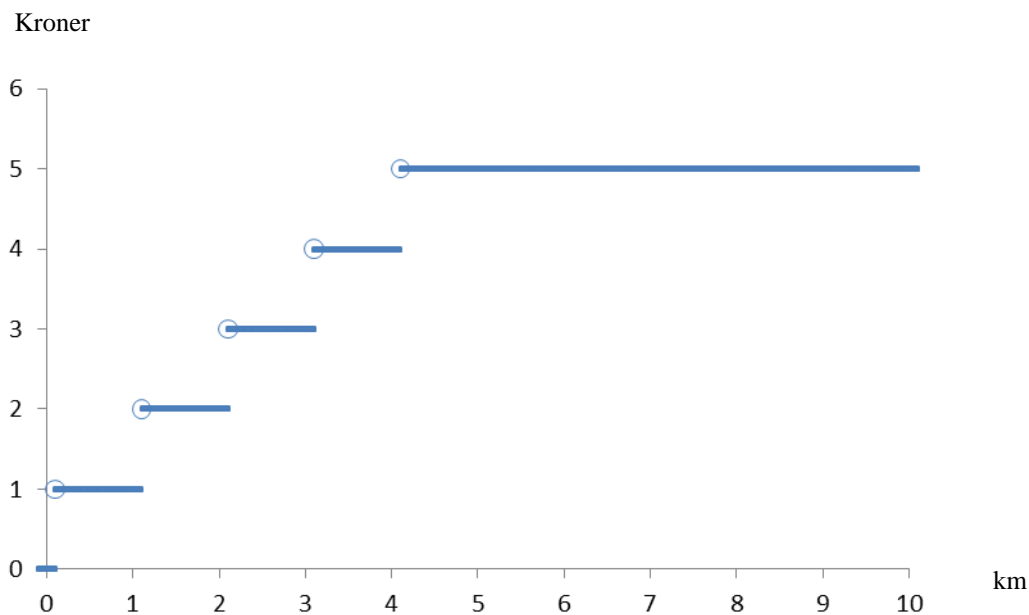
Kroner
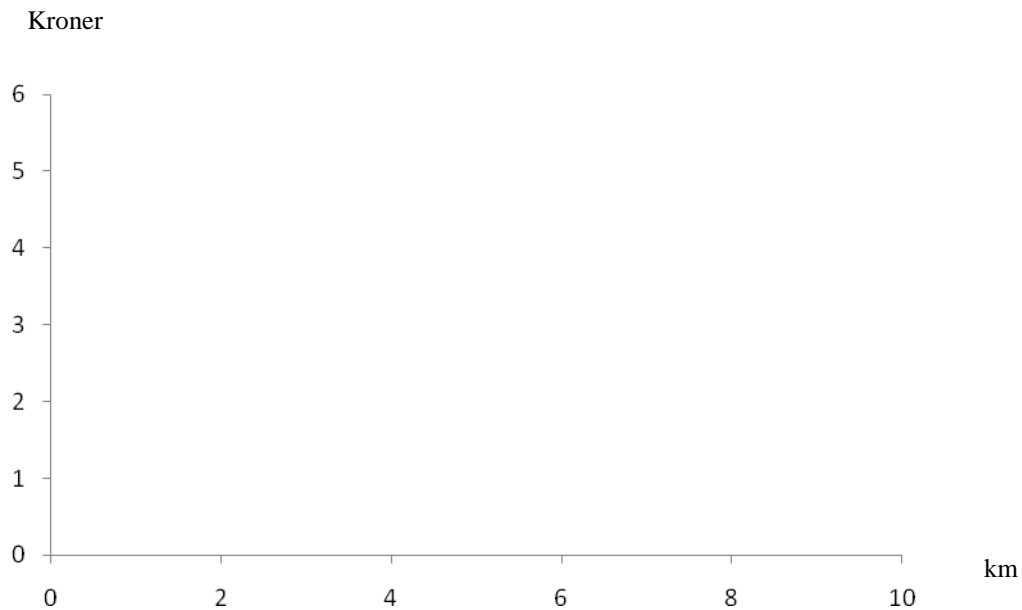
Exercise SPM 5-3

The new city council of Aakebrö decrees a different pricing system for the trolleybuses. That system is shown by the graph below. Explanations:

- Bold dots belong to the graph, empty dots don't.
- The *x*-axis shows values up to *x*=10, but the function extends to values beyond 10.

State the assignment of this new pricing system, that is a function *C* showing transportation cost per day [in kroners] as a function of travelling distance [in km] per day for one person.

SOLUTIONS

Exercise SPM 1-1

a) i) $D\setminus B = D \cap \bar{B}$

ii) $D \cap B \cap A$

iii) $B \cap D\setminus A = B \cap D \cap \bar{A}$

b) iv) $B \cap C$ is the set of all women aged at least 10 years.

v) $A \cap D\setminus B$ is the set of all retired men having bloodgroup A

Exercise SPM 2-1

a) true

b) true

c) false

d) true

e) false

f) true

Exercise SPM 2–2

a) true

b) false.

c) false

Exercise SPM 3-1

The assertion is true for $x = 9$ and for $x = \pi$. It is false for $x = 10$.

Exercise SPM 3-1

The assertion is true for $x = 101$. It is false for $x = 100$ and for $x = 300 + \pi$.

Exercise SPM 4.1-1

a) $x(wz - av) = xwz - xav$

b) $(34x + 17y)y = 34xy + 17y^2$

Exercise SPM 4.1-2

a) $abx + avx = ax(b + v)$

b) $5xy - 10xz = 5x(y - 2z)$

Exercise SPM 4.1-3

a) $\dfrac{ab}{c} - \dfrac{cd}{a} = \dfrac{aba - cdc}{ac} = \dfrac{a^2 b - c^2 d}{ac}$

b) $\dfrac{2(a+b)}{3c} + \dfrac{3(a-b)}{2a} = \dfrac{2(a+b)2a + 3(a-b)3c}{6ac} = \dfrac{4a(a+b) + 9c(a-b)}{6ac}$

Exercise SPM 4.1-4

a) $\dfrac{\dfrac{4c}{ab}}{\dfrac{3d}{c^2}} = \dfrac{4cc^2}{ab3d} = \dfrac{4c^3}{3abd}$

b) $\dfrac{\dfrac{2(a+b)}{3c}}{\dfrac{3(a-b)}{2a}} = \dfrac{2(a+b)2a}{3c3(a-b)} = \dfrac{4a(a+b)}{9c(a-b)}$

Exercise SPM 4.1–5

a) $(x-1)^2 = x^2 - 2x + 1$

b) $(z+1)^3 = z^3 + 3z^2 + 3z + 1$

Exercise SPM 4.1-6

a) $(x-1)^2(x+1)^2 = [(x-1)(x+1)]^2 = (x^2-1)^2$

b) $\dfrac{(z+1)^3}{(z-1)^3} = \left[\dfrac{z+1}{z-1}\right]^3$

Exercise SPM 4.1-6

a) $1300\ mm^2 = 0{,}0013\ m^2$

b) $23\ \dfrac{m}{\sec} = 3{,}6 \cdot 23\ \dfrac{km}{h} = 82{,}8\ \dfrac{km}{h}$

c) $45\ m^3 = 45'000'000\ cm^3$

Exercise SPM 4.1-7

a) $2\log_a x + \log_a y = \log_a(x^2) + \log_a y = \log_a(x^2 y)$

b) $3\log_a x - \dfrac{1}{3}\log_a x = \log_a(x^3) - \log_a(x^{\frac{1}{3}}) = \log_a\left(\dfrac{x^3}{x^{\frac{1}{3}}}\right) = \log_a(x^{\frac{8}{3}}) = \log_a(\sqrt[3]{x^8})$

Exercise SPM 4.2-1

a) $(x-1)(x+1) = 0 \quad\Leftrightarrow\quad x = 1 \quad$ or $\quad x = -1$. These equations are equivalent.

b) $x^2 + x - 2 = 0 \Leftarrow \quad x = 1$. The inverse implication would be false, because $x = -2$ is another solution of the left side equation. By consequence, the equations are not equivalent.

Exercise SPM 4.3-1

$\dfrac{\log_{10} y}{(x^2-1)} - 5 = 7 \Leftrightarrow \dfrac{\log_{10} y}{(x^2-1)} = 7 + 5 = 12 \Leftrightarrow \log_{10} y = 12(x^2-1) \Leftrightarrow y = 10^{12(x^2-1)}$

Exercise SPM 4.3-2

$(x^2 - 3x + 9)(x^2 - 1) = 0 \Leftrightarrow x^2 - 3x + 9 = 0$ or $x^2 - 1 = 0 \Leftrightarrow$

$(x-1)(x+1) = 0 \Leftrightarrow x \in \{1,\ -1\}$

Explanation: the quadratic equation $x^2 - 3x + 9 = 0$ has no real solution.

Exercise SPM 4.3-3

$$\frac{(x-1)(x+9)(x-9)}{(x-9)(x-17)} = 0 \Leftrightarrow (x-1)(x+9)(x-9) = 0 \text{ and } (x-9)(x-17) \neq 0$$

$$\Leftrightarrow x \in \{1, -9, 9\} \text{ and } x \notin \{9, 17\} \Leftrightarrow x \in \{1, -9\}$$

Exercise SPM 4.3-4

$$2^y = 2^{y+1} - 64 = 2^{y+1} - 2^6 \Leftrightarrow 2^6 = 2^{y+1} - 2^y = 2^y(2-1) = 2^y \Leftrightarrow y = 6$$

Exercise SPM 4.3-5

$$(x-3)^4 = 16 \Leftrightarrow (x-3)^2 = 4 \Leftrightarrow x-3 = 2 \text{ or } x-3 = -2 \Leftrightarrow x \in \{5, 1\}$$

Exercise SPM 4.3-6

$$2^{3x-5} = 4^{5x} = (2^2)^{5x} = 2^{2 \cdot 5x} = 2^{10x} \Leftrightarrow 3x-5 = 10x \Leftrightarrow -5 = 7x \Leftrightarrow x = -\frac{5}{7}$$

Exercise SPM 4.4-1

a) Exactly one solution (which is 1).
b) Exactly two solutions (0 and 2).
c) No real solution (negative discriminant).

Exercise SPM 4.4-2

$y(0) = 2 = b, \quad y(2) = 8 = 2a + 2 \Leftrightarrow 2a = 6 \Leftrightarrow a = 3$, so we have $y = 3x + 2$.

Verification: $3 \cdot 0 + 2 = 2$ and $3 \cdot 2 + 2 = 8$.

Exercise SPM 4.5-1

Multiply the second equation with 2 and add it to the first equation to obtain $9y = 5$, that is

$y = \frac{5}{9}$. The equation $x + 4y = 2$ now implies

$$x = 2 - 4y = 2 - 4 \cdot \frac{5}{9} = \frac{2 \cdot 9 - 4 \cdot 5}{9} = \frac{18 - 20}{9} = -\frac{2}{9}.$$

Verification: $\begin{cases} -2 \cdot (-\frac{2}{9}) + \frac{5}{9} = \frac{9}{9} = 1 \\ -\frac{2}{9} + 4\frac{5}{9} = \frac{18}{9} = 2 \end{cases}$

Exercise SPM 4.5-2

Since $y^2 = -22$ has no real solution, the entire equation system has no real solution.

Exercise SPM 4.5-3

$x + z = 1$ implies $z = 1 - x$. Inserting this in the second equation yields
$y + 3(1-x) = y + 3 - 3x = 2$, so $y - 3x = -1$.
We now have the equation system
$$\begin{cases} 2x + 3y = 1 \\ -3x + y = -1 \end{cases}$$

This implies $2x + 3y = 3x - y$, and, by consequence, $4y = x$.

Inserting this in the first equation yields $2 \cdot 4y + 3y = 11y = 1$, which means $y = \dfrac{1}{11}$. We now

obtain $x = \dfrac{4}{11}$ and $z = 1 - \dfrac{4}{11} = \dfrac{7}{11}$.

Verification: $\begin{cases} 2 \cdot \dfrac{4}{11} + 3 \cdot \dfrac{1}{11} = \dfrac{11}{11} = 1 \\[2mm] \dfrac{1}{11} + 3 \cdot \dfrac{7}{11} = \dfrac{22}{11} = 2 \\[2mm] \dfrac{4}{11} + \dfrac{7}{11} = \dfrac{11}{11} = 1 \end{cases}$

Exercise SPM 5-1
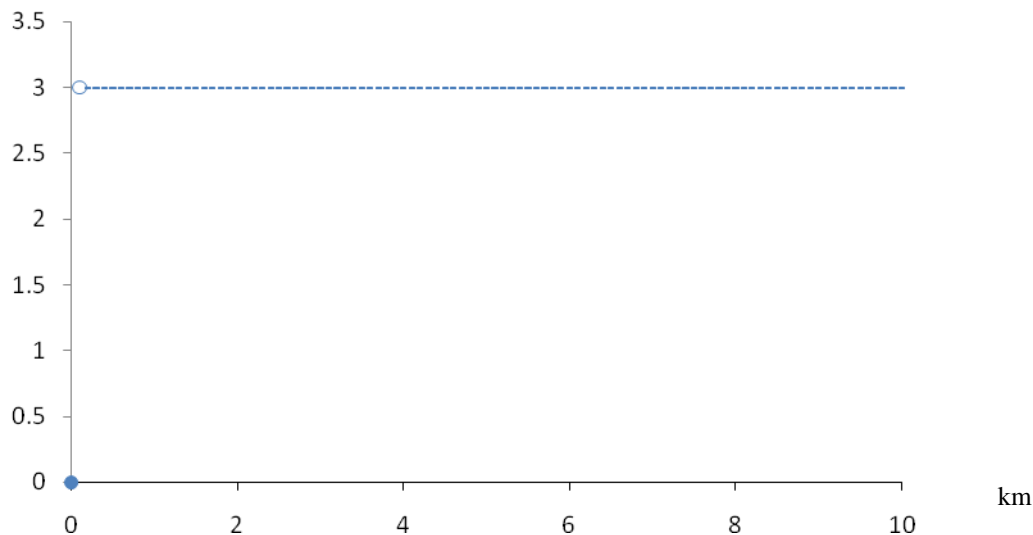The radicand must be nonnegative, which means $x \geq 3$.

Exercise SPM 5-2
a) $D_C = \left[0, \ \infty \right)$

b) $C(x) = \begin{cases} 0 & \text{for } x = 0 \\ 3 & \text{for } x > 0 \end{cases}$

c) Explanation: solid points are part of the graph, empty points are not.

Kroner

Exercise SPM 5-3

$$C(x) = \begin{cases} 0 & \text{for } x = 0 \\ 1 & \text{for } 0 < x \le 1 \\ 2 & \text{for } 1 < x \le 2 \\ 3 & \text{for } 2 < x \le 3 \\ 4 & \text{for } 3 < x \le 4 \\ 5 & \text{for } x > 4 \end{cases}$$